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Some coupled fixed-point theorems in two quasi-partial metric spaces

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Abstract
The purpose of this paper is to prove some new coupled common fixed-point theorems for mappings defined on a set equipped with two quasi-partial metrics. We also provide illustrative examples in support of our new results.
MSC: 47H10; 54H25

Keywords: common coupled fixed point; coupled coincidence point; *w*-compatible mapping pairs; quasi-partial metric space

1 Introduction and preliminaries

In 1994, Matthews [1] introduced the notion of partial metric spaces as follows.

Definition 1.1 [1] A *partial metric* on a nonempty set X is a function $p : X \times X \longrightarrow \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
(p2) $p(x,x) \leq p(x,y)$,
(p3) $p(x,y) = p(y,x)$,
(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

In [1], Matthews extended the Banach contraction principle from metric spaces to partial metric spaces. Based on the notion of partial metric spaces, several authors (for example, [2–32]) obtained some fixed-point results for mappings satisfying different contractive conditions. Very recently, Haghi *et al.* [33] showed in their interesting paper that some fixed-point theorems in partial metric spaces can be obtained from metric spaces.

Karapınar *et al.* [34] introduced the concept of quasi-partial metric spaces and studied some fixed-point problems on quasi-partial metric spaces. The notion of a quasi-partial metric space is defined as follows.

Definition 1.2 [34] A *quasi-partial metric* on nonempty set X is a function $q : X \times X \to \mathbb{R}^+$ which satisfies:

(QPM$_1$) If $q(x,x) = q(x,y) = q(y,y)$, then $x = y$,
(QPM$_2$) $q(x,x) \leq q(x,y)$,

(QPM_3) $q(x,x) \leq q(y,x)$, and
(QPM_4) $q(x,y) + q(z,z) \leq q(x,z) + q(z,y)$

for all $x, y, z \in X$.

A *quasi-partial metric space* is a pair (X,q) such that X is a nonempty set and q is a quasi-partial metric on X.

Let q be a quasi-partial metric on set X. Then

$$d_q(x,y) = q(x,y) + q(y,x) - q(x,x) - q(y,y)$$

is a metric on X.

Definition 1.3 [34] Let (X,q) be a quasi-partial metric space. Then

(i) A sequence $\{x_n\}$ *converges* to a point $x \in X$ if and only if

$$q(x,x) = \lim_{n\to\infty} q(x,x_n) = \lim_{n\to\infty} q(x_n,x).$$

(ii) A sequence $\{x_n\}$ is called a *Cauchy sequence* if $\lim_{n,m\to\infty} q(x_n,x_m)$ and $\lim_{n,m\to\infty} q(x_m,x_n)$ exist (and are finite).

(iii) The quasi-partial metric space (X,q) is said to be *complete* if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_q, to a point $x \in X$ such that

$$q(x,x) = \lim_{n,m\to\infty} q(x_n,x_m) = \lim_{n,m\to\infty} q(x_n,x_m).$$

Bhaskar and Lakshmikantham [35] introduced the concept of a coupled fixed point and studied some nice coupled fixed-point theorems. Later, Lakshmikantham and Ćirić [36] introduced the notion of a coupled coincidence point of mappings. For some works on a coupled fixed point, we refer the reader to [37–62].

Definition 1.4 [35] Let X be a nonempty set. We call an element $(x,y) \in X \times X$ a *coupled fixed point* of the mapping $F: X \times X \to X$ if $F(x,y) = x$ and $F(y,x) = y$.

Definition 1.5 [36] An element $(x,y) \in X \times X$ is called

(i) a *coupled coincidence point* of the mapping $F: X \times X \to X$ and $g: X \to X$ if $F(x,y) = gx$ and $F(y,x) = gy$; in this case (gx,gy) is called *coupled point of coincidence* of mappings F and g;

(ii) a *common coupled fixed point* of mappings $F: X \times X \to X$ and $g: X \to X$ if $F(x,y) = gx = x$ and $F(y,x) = gy = y$;

(iii) a *common coupled fixed point* of mappings $F: X \times X \to X$ and $g: X \to X$ if $F(x,y) = gx = x$ and $F(y,x) = gy = y$.

Abbas *et al.* [37] introduced the concept of w-compatible mappings as follows.

Definition 1.6 [37] Let X be a nonempty set. We say that the mappings $F: X \times X \to X$ and $g: X \to X$ are *w-compatible* if $gF(x,y) = F(gx,gy)$ whenever $gx = F(x,y)$ and $gy = F(y,x)$.

Very recently, Shatanawi and Pitea [38] obtained some common coupled fixed-point results for a pair of mappings in quasi-partial metric space.

Theorem 1.1 (see [38, Theorem 2.1]) *Let (X,q) be a quasi-partial metric space, $g:X\to X$ and $F:X\times X\to X$ be two mappings. Suppose that there exist k_1, k_2, and k_3 in $[0,1)$ with $k_1+k_2+k_3<1$ such that the condition*

$$\begin{aligned} &q\bigl(F(x,y),F(u,v)\bigr)+q\bigl(F(y,x),F(v,u)\bigr)\\ &\quad\le k_1\bigl[q(gx,gu)+q(gy,gv)\bigr]+k_2\bigl[q\bigl(gx,F(x,y)\bigr)+q\bigl(gy,F(y,x)\bigr)\bigr]\\ &\qquad+k_3\bigl[q\bigl(gu,F(u,v)\bigr)+q\bigl(gv,F(v,u)\bigr)\bigr] \end{aligned} \tag{1.1}$$

holds for all $x,y,u,v\in X$. Also, suppose we have the following hypotheses:
(i) *$F(X\times X)\subset g(X)$.*
(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q.*
Then the mappings F and g have a coincidence point (x,y) satisfying $gx=F(x,y)$ and $gy=F(y,x)$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (x,x).

The aim of this article is to prove some new coupled common fixed-point theorems for mappings defined on a set equipped with two quasi-partial metrics.

The following lemma is crucial in our work.

Lemma 1.1 [38] *Let (X,q) be a quasi-partial metric space. Then the following statements hold true*:
(i) *If $q(x,y)=0$, then $x=y$.*
(ii) *If $x\neq y$, then $q(x,y)>0$ and $q(y,x)>0$.*

In this manuscript, we generalize, improve, enrich, and extend the above coupled common fixed-point results. We also state some examples to illustrate our results. This paper can be considered as a continuation of the remarkable works of Aydi [12], Karapınar *et al.* [34], and Shatanawi and Pitea [38].

2 Main results

Now we shall prove our main results.

Theorem 2.1 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and let $F:X\times X\to X$, $g:X\to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with*

$$k_1+k_2+k_3+2k_4+k_5<1 \tag{2.1}$$

such that the condition

$$\begin{aligned} &q_1\bigl(F(x,y),F(u,v)\bigr)+q_1\bigl(F(y,x),F(v,u)\bigr)\\ &\quad\le k_1\bigl[q_2(gx,gu)+q_2(gy,gv)\bigr]+k_2\bigl[q_2\bigl(gx,F(x,y)\bigr)+q_2\bigl(gy,F(y,x)\bigr)\bigr] \end{aligned}$$

$$\begin{aligned} &+ k_3\big[q_2\big(gu, F(u,v)\big) + q_2\big(gv, F(v,u)\big)\big] + k_4\big[q_2\big(gx, F(u,v)\big) + q_2\big(gy, F(v,u)\big)\big] \\ &+ k_5\big[q_2\big(gu, F(x,y)\big) + q_2\big(gv, F(y,x)\big)\big] \end{aligned} \tag{2.2}$$

holds for all $x, y, u, v \in X$. *Also, suppose we have the following hypotheses*:

(i) $F(X \times X) \subset g(X)$.

(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial metric* q_1.

Then the mappings F *and* g *have a coincidence point* (x, y) *satisfying* $gx = F(x,y) = F(y,x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof Let $x_0, y_0 \in X$. Since $F(X \times X) \subset g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$. Similarly, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. Continuing in this way we construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$gx_{n+1} = F(x_n, y_n) \quad \text{and} \quad gy_{n+1} = F(y_n, x_n), \quad \forall n \geq 0. \tag{2.3}$$

It follows from (2.2) and (QPM_4) that

$$\begin{aligned} &q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1}) \\ &\quad = q_1\big(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big) + q_1\big(F(y_{n-1}, x_{n-1}), F(y_n, x_n)\big) \\ &\quad \leq k_1\big[q_2(gx_{n-1}, gx_n) + q_2(gy_{n-1}, gy_n)\big] \\ &\qquad + k_2\big[q_2\big(gx_{n-1}, F(x_{n-1}, y_{n-1}) + q_2\big(gy_{n-1}, F(y_{n-1}, x_{n-1})\big)\big)\big] \\ &\qquad + k_3\big[q_2\big(gx_n, F(x_n, y_n)\big) + q_2\big(gy_n, F(y_n, x_n)\big)\big] \\ &\qquad + k_4\big[q_2\big(gx_{n-1}, F(x_n, y_n)\big) + q_2\big(gy_{n-1}, F(y_n, x_n)\big)\big] \\ &\qquad + k_5\big[q_2\big(gx_n, F(x_{n-1}, y_{n-1})\big) + q_2\big(gy_n, F(y_{n-1}, x_{n-1})\big)\big] \\ &\quad = (k_1 + k_2)\big[q_2(gx_{n-1}, gx_n) + q_2(gy_{n-1}, gy_n)\big] + k_3\big[q_2(gx_n, gx_{n+1}) + q_2(gy_n, gy_{n+1})\big] \\ &\qquad + k_4\big[q_2(gx_{n-1}, gx_{n+1}) + q_2(gy_{n-1}, gy_{n+1})\big] + k_5\big[q_2(gx_n, gx_n) + q_2(gy_n, gy_n)\big] \\ &\quad \leq (k_1 + k_2)\big[q_2(gx_{n-1}, gx_n) + q_2(gy_{n-1}, gy_n)\big] + k_3\big[q_2(gx_n, gx_{n+1}) + q_2(gy_n, gy_{n+1})\big] \\ &\qquad + k_4\big[q_2(gx_{n-1}, gx_n) + q_2(gx_n, gx_{n+1}) - q_2(gx_n, gx_n) + q_2(gy_{n-1}, gy_n) + q_2(gy_n, gy_{n+1}) \\ &\qquad - q_2(gy_n, gy_n)\big] + k_5\big[q_2(gx_n, gx_{n+1}) + q_2(gy_n, gy_{n+1})\big] \\ &\quad \leq (k_1 + k_2 + k_4)\big[q_2(gx_{n-1}, gx_n) + q_2(gy_{n-1}, gy_n)\big] \\ &\qquad + (k_3 + k_4 + k_5)\big[q_2(gx_n, gx_{n+1}) + q_2(gy_n, gy_{n+1})\big] \\ &\quad \leq (k_1 + k_2 + k_4)\big[q_1(gx_{n-1}, gx_n) + q_1(gy_{n-1}, gy_n)\big] \\ &\qquad + (k_3 + k_4 + k_5)\big[q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1})\big], \end{aligned}$$

which implies that

$$q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1}) \le \frac{k_1 + k_2 + k_4}{1 - k_3 - k_4 - k_5}\big[q_1(gx_{n-1}, gx_n) + q_1(gy_{n-1}, gy_n)\big]. \tag{2.4}$$

Put $k = \frac{k_1+k_2+k_4}{1-k_3-k_4-k_5}$. Obviously, $0 \le k < 1$. By repetition of the above inequality (2.4) n times, we get

$$q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1}) \le k^n\big[q_1(gx_0, gx_1) + q_1(gy_0, gy_1)\big]. \tag{2.5}$$

Next, we shall prove that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in $g(X)$.

In fact, for each $n, m \in \mathbb{N}$, $m > n$, from (QPM_4) and (2.5) we have

$$\begin{aligned} q_1(gx_n, gx_m) + q_1(gy_n, gy_m) &\le \sum_{i=n}^{m-1}\big[q_1(gx_i, gx_{i+1}) + q_1(gy_i, gy_{i+1})\big] \\ &\le \sum_{i=n}^{m-1} k^i\big[q_1(gx_0, gx_1) + q_1(gy_0, gy_1)\big] \\ &\le \frac{k^n}{1-k}\big[q_1(gx_0, gx_1) + q_1(gy_0, gy_1)\big]. \end{aligned} \tag{2.6}$$

This implies that

$$\lim_{n,m\to\infty}\big[q_1(gx_n, gx_m) + q_1(gy_n, gy_m)\big] = 0,$$

and so

$$\lim_{n,m\to\infty} q_1(gx_n, gx_m) = 0 \quad \text{and} \quad \lim_{n,m\to\infty} q_1(gy_n, gy_m) = 0. \tag{2.7}$$

By similar arguments as above, we can show that

$$\lim_{n,m\to\infty} q_1(gx_m, gx_n) = 0 \quad \text{and} \quad \lim_{n,m\to\infty} q_1(gy_m, gy_n) = 0. \tag{2.8}$$

Hence $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in (gX, q_1). Since (gX, q_1) is complete, there exist $gx, gy \in g(X)$ such that $\{gx_n\}$ and $\{gy_n\}$ converge to gx and gy with respect to τ_{q_1}, that is,

$$\begin{aligned} q_1(gx, gx) &= \lim_{n\to\infty} q_1(gx, gx_n) = \lim_{n\to\infty} q_1(gx_n, gx) \\ &= \lim_{n,m\to\infty} q_1(gx_m, gx_n) = \lim_{n,m\to\infty} q_1(gx_n, gx_m) \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} q_1(gy, gy) &= \lim_{n\to\infty} q_1(gy, gy_n) = \lim_{n\to\infty} q_1(gy_n, gy) \\ &= \lim_{n,m\to\infty} q_1(gy_m, gy_n) = \lim_{n,m\to\infty} q_1(gy_n, gy_m). \end{aligned} \tag{2.10}$$

Combining (2.7)-(2.10), we have

$$\begin{aligned} q_1(gx,gx) &= \lim_{n\to\infty} q_1(gx,gx_n) = \lim_{n\to\infty} q_1(gx_n,gx) \\ &= \lim_{n,m\to\infty} q_1(gx_m,gx_n) = \lim_{n,m\to\infty} q_1(gx_n,gx_m) = 0 \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} q_1(gy,gy) &= \lim_{n\to\infty} q_1(gy,gy_n) = \lim_{n\to\infty} q_1(gy_n,gy) \\ &= \lim_{n,m\to\infty} q_1(gy_m,gy_n) = \lim_{n,m\to\infty} q_1(gy_n,gy_m) = 0. \end{aligned} \tag{2.12}$$

By (QPM_4) we obtain

$$\begin{aligned} q_1\bigl(gx_{n+1},F(x,y)\bigr) &\le q_1(gx_{n+1},gx) + q_1\bigl(gx,F(x,y)\bigr) - q_1(gx,gx) \\ &\le q_1(gx_{n+1},gx) + q_1\bigl(gx,F(x,y)\bigr) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1\bigl(gx_{n+1},F(x,y)\bigr) - q_1(gx_{n+1},gx_{n+1}) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1\bigl(gx_{n+1},F(x,y)\bigr). \end{aligned}$$

Letting $n \to \infty$ in the above inequalities and using (2.11), we have

$$\lim_{n\to\infty} q_1\bigl(gx_{n+1},F(x,y)\bigr) \le q_1\bigl(gx,F(x,y)\bigr) \le \lim_{n\to\infty} q_1\bigl(gx_{n+1},F(x,y)\bigr).$$

That is,

$$\lim_{n\to\infty} q_1\bigl(gx_{n+1},F(x,y)\bigr) = q_1\bigl(gx,F(x,y)\bigr). \tag{2.13}$$

Similarly, using (2.12) we have

$$\lim_{n\to\infty} q_1\bigl(gy_{n+1},F(y,x)\bigr) = q_1\bigl(gy,F(y,x)\bigr). \tag{2.14}$$

Now we prove that $F(x,y) = gx$ and $F(y,x) = gy$. In fact, it follows from (2.2) and (2.3) that

$$\begin{aligned} &q_1\bigl(gx_{n+1},F(x,y)\bigr) + q_1\bigl(gy_{n+1},F(y,x)\bigr) \\ &\quad = q_1\bigl(F(x_n,y_n),F(x,y)\bigr) + q_1\bigl(F(y_n,x_n)\bigr) \\ &\quad \le k_1\bigl[q_2(gx_n,gx) + q_2(gy_n,gy)\bigr] + k_2\bigl[q_2\bigl(gx_n,F(x_n,y_n)\bigr) + q_2\bigl(gy_n,F(y_n,x_n)\bigr)\bigr] \\ &\qquad + k_3\bigl[q_2\bigl(gx,F(x,y)\bigr) + q_2\bigl(gy,F(y,x)\bigr)\bigr] + k_4\bigl[q_2\bigl(gx_n,F(x,y)\bigr) + q_2\bigl(gy_n,F(y,x)\bigr)\bigr] \\ &\qquad + k_5\bigl[q_2\bigl(gx,F(x_n,y_n)\bigr) + q_2\bigl(gy,F(y_n,x_n)\bigr)\bigr] \\ &\quad = k_1\bigl[q_2(gx_n,gx) + q_2(gy_n,gy)\bigr] + k_2\bigl[q_2(gx_n,gx_{n+1}) + q_2(gy_n,gy_{n+1})\bigr] \\ &\qquad + k_3\bigl[q_2\bigl(gx,F(x,y)\bigr) + q_2\bigl(gy,F(y,x)\bigr)\bigr] + k_4\bigl[q_2\bigl(gx_n,F(x,y)\bigr) + q_2\bigl(gy_n,F(y,x)\bigr)\bigr] \\ &\qquad + k_5\bigl[q_2(gx,gx_{n+1}) + q_2(gy,gy_{n+1})\bigr] \\ &\quad \le k_1\bigl[q_1(gx_n,gx) + q_1(gy_n,gy)\bigr] + k_2\bigl[q_1(gx_n,gx_{n+1}) + q_1(gy_n,gy_{n+1})\bigr] \end{aligned}$$

$$+ k_3\big[q_1\big(gx, F(x,y)\big) + q_1\big(gy, F(y,x)\big)\big] + k_4\big[q_1\big(gx_n, F(x,y)\big) + q_1\big(gy_n, F(y,x)\big)\big]$$
$$+ k_5\big[q_1(gx, gx_{n+1}) + q_1(gy, gy_{n+1})\big].$$

Letting $n \to \infty$ in the above inequality, using (2.11)-(2.14), we obtain

$$q_1\big(gx, F(x,y)\big) + q_1\big(gy, F(y,x)\big) \le (k_3 + k_4)\big[q_1\big(gx, F(x,y)\big) + q_1\big(gy, F(y,x)\big)\big]. \tag{2.15}$$

By (2.1) we have $k_3 + k_4 < 1$. Hence, it follows from (2.15) that $q_1(gx, F(x,y)) = q_1(gy, F(y,x)) = 0$. By Lemma 1.1, we get $F(x,y) = gx$ and $F(y,x) = gy$. Hence, (gx, gy) is a coupled point of coincidence of mappings F and g.

Next, we will show that the coupled point of coincidence is unique. Suppose that $(x^*, y^*) \in X \times X$ with $F(x^*, y^*) = gx^*$ and $F(y^*, x^*) = gy^*$. Using (2.2), (2.11), (2.12), and (QPM_3), we obtain

$$\begin{aligned}
&q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big) \\
&\quad = q_1\big(F(x,y), F\big(x^*, y^*\big)\big) + q_1\big(F(y,x), F\big(y^*, x^*\big)\big) \\
&\quad \le k_1\big[q_2\big(gx, gx^*\big) + q_2\big(gy, gy^*\big)\big] + k_2\big[q_2\big(gx, F(x,y)\big) + q_2\big(gy, F(y,x)\big)\big] \\
&\qquad + k_3\big[q_2\big(gx^*, F\big(x^*, y^*\big)\big) + q_2\big(gy^*, F\big(y^*, x^*\big)\big)\big] \\
&\qquad + k_4\big[q_2\big(gx, F\big(x^*, y^*\big)\big) + q_2\big(gy, F\big(y^*, x^*\big)\big)\big] \\
&\qquad + k_5\big[q_2\big(gx^*, F(x,y)\big) + q_2\big(gy^*, F(y,x)\big)\big] \\
&\quad = k_1\big[q_2\big(gx, gx^*\big) + q_2\big(gy, gy^*\big)\big] + k_2\big[q_2(gx, gx) + q_2(gy, gy)\big] \\
&\qquad + k_3\big[q_2\big(gx^*, gx^*\big) + q_2\big(gy^*, gy^*\big)\big] + k_4\big[q_2\big(gx, gx^*\big) + q_2\big(gy, gy^*\big)\big] \\
&\qquad + k_5\big[q_2\big(gx^*, gx\big) + q_2\big(gy^*, gy\big)\big] \\
&\quad \le (k_1 + k_4)\big[q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big)\big] + k_2\big[q_1(gx, gx) + q_1(gy, gy)\big] \\
&\qquad + k_3\big[q_1\big(gx^*, gx^*\big) + q_1\big(gy^*, gy^*\big)\big] + k_5\big[q_1\big(gx^*, gx\big) + q_1\big(gy^*, gy\big)\big] \\
&\quad \le (k_1 + k_3 + k_4)\big[q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big)\big] \\
&\qquad + k_5\big[q_1\big(gx^*, gx\big) + q_1\big(gy^*, gy\big)\big].
\end{aligned}$$

This implies that

$$q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big) \le \frac{k_5}{1 - k_1 - k_3 - k_4} \cdot \big[q_1\big(gx^*, gx\big) + q_1\big(gy^*, gy\big)\big]. \tag{2.16}$$

Similarly, we have

$$q_1\big(gx^*, gx\big) + q_1\big(gy^*, gy\big) \le \frac{k_5}{1 - k_1 - k_3 - k_4} \cdot \big[q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big)\big]. \tag{2.17}$$

Substituting (2.17) into (2.16), we obtain

$$q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big) \le \left(\frac{k_5}{1 - k_1 - k_3 - k_4}\right)^2 \cdot \big[q_1\big(gx, gx^*\big) + q_1\big(gy, gy^*\big)\big]. \tag{2.18}$$

Since $\frac{k_5}{1-k_1-k_3-k_4} < 1$, from (2.18), we must have $q_1(gx, gx^*) = q_1(gy, gy^*) = 0$. By Lemma 1.1, we get $gx = gx^*$ and $gy = gy^*$, which implies the uniqueness of the coupled point of coincidence of F and g, that is, (gx, gy).

Next, we will show that $gx = gy$. In fact, from (2.2), (2.11), and (2.12) we have

$$\begin{aligned}
&q_1(gx,gy) + q_1(gy,gx) \\
&\quad = q_1\bigl(F(x,y),F(y,x)\bigr) + q_1\bigl(F(y,x),F(x,y)\bigr) \\
&\quad \le k_1\bigl[q_2(gx,gy) + q_2(gy,gx)\bigr] + k_2\bigl[q_2\bigl(gx,F(x,y)\bigr) + q_2\bigl(gy,F(y,x)\bigr)\bigr] \\
&\qquad + k_3\bigl[q_2\bigl(gy,F(y,x)\bigr) + q_2\bigl(gx,F(x,y)\bigr)\bigr] + k_4\bigl[q_2\bigl(gx,F(y,x)\bigr) + q_2\bigl(gy,F(x,y)\bigr)\bigr] \\
&\qquad + k_5\bigl[q_2\bigl(gy,F(x,y)\bigr) + q_2\bigl(gx,F(y,x)\bigr)\bigr] \\
&\quad = k_1\bigl[q_2(gx,gy) + q_2(gy,gx)\bigr] + k_2\bigl[q_2(gx,gx) + q_2(gy,gy)\bigr] \\
&\qquad + k_3\bigl[q_2(gy,gy) + q_2(gx,gx)\bigr] + k_4\bigl[q_2(gx,gy) + q_2(gy,gx)\bigr] \\
&\qquad + k_5\bigl[q_2(gy,gx) + q_2(gx,gy)\bigr] \\
&\quad \le k_1\bigl[q_1(gx,gy) + q_1(gy,gx)\bigr] + k_2\bigl[q_1(gx,gx) + q_1(gy,gy)\bigr] \\
&\qquad + k_3\bigl[q_1(gy,gy) + q_1(gx,gx)\bigr] + k_4\bigl[q_1(gx,gy) + q_1(gy,gx)\bigr] \\
&\qquad + k_5\bigl[q_1(gy,gx) + q_1(gx,gy)\bigr] \\
&\quad = (k_1 + k_4 + k_5)\bigl[q_1(gx,gy) + q_1(gy,gx)\bigr].
\end{aligned} \tag{2.19}$$

Since $k_1 + k_4 + k_5 < 1$, we have $q_1(gx, gy) = q_1(gy, gx) = 0$. By Lemma 1.1, we get $gx = gy$.

Finally, assume that g and F are w-compatible. Let $u = gx$, then we have $u = gx = F(x, y) = gy = F(y, x)$, so that

$$gu = ggx = g\bigl(F(x,y)\bigr) = F(gx,gy) = F(u,u). \tag{2.20}$$

Consequently, (u, u) is a coupled coincidence point of F and g, and therefore (gu, gu) is a coupled point of coincidence of F and g, and by its uniqueness, we get $gu = gx$. Thus, we obtain $F(u, u) = gu = u$. Therefore, (u, u) is the unique common coupled fixed point of F and g. This completes the proof of Theorem 2.1. □

In Theorem 2.1, if we take $q_1(x, y) = q_2(x, y)$ for all $x, y \in X$, then we get the following.

Corollary 2.1 *Let (X, q) be a quasi-partial metric space, $F : X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4 and k_5 in $[0, 1)$ with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition*

$$\begin{aligned}
&q\bigl(F(x,y),F(u,v)\bigr) + q\bigl(F(y,x),F(v,u)\bigr) \\
&\quad \le k_1\bigl[q(gx,gu) + q(gy,gv)\bigr] + k_2\bigl[q\bigl(gx,F(x,y)\bigr) + q\bigl(gy,F(y,x)\bigr)\bigr] \\
&\qquad + k_3\bigl[q\bigl(gu,F(u,v)\bigr) + q\bigl(gv,F(v,u)\bigr)\bigr] + k_4\bigl[q\bigl(gx,F(u,v)\bigr) + q\bigl(gy,F(v,u)\bigr)\bigr] \\
&\qquad + k_5\bigl[q\bigl(gu,F(x,y)\bigr) + q\bigl(gv,F(y,x)\bigr)\bigr]
\end{aligned} \tag{2.21}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X \times X) \subset g(X)$.
(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial metric* q.
Then the mappings F *and* g *have a coincidence point* (x,y) *satisfying* $gx = F(x,y) = F(y,x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Remark 2.1 Corollary 2.1 improve and extend Theorem 2.1 of Shatanawi and Pitea [38]; the contractive condition defined by (1.1) is replaced by the new contractive condition defined by (2.23).

Corollary 2.2 *Let* q_1 *and* q_2 *be two quasi-metrics on* X *such that* $q_2(x,y) \leq q_1(x,y)$, *for all* $x,y \in X$, *and* $F : X \times X \to X$, $g : X \to X$ *be two mappings. Suppose that there exist* $a_i \in [0,1)$ $(i = 1,2,3,\dots,10)$ *with*

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + 2(a_7 + a_8) + a_9 + a_{10} < 1 \tag{2.22}$$

such that the condition

$$\begin{aligned} &q_1\bigl(F(x,y),F(u,v)\bigr) \\ &\quad \leq a_1 q_2(gx,gu) + a_2 q_2(gy,gv) + a_3 q_2\bigl(gx,F(x,y)\bigr) + a_4 q_2\bigl(gy,F(y,x)\bigr) \\ &\qquad + a_5 q_2\bigl(gu,F(u,v)\bigr) + a_6 q_2\bigl(gv,F(v,u)\bigr) + a_7 q_2\bigl(gx,F(u,v)\bigr) + a_8 q_2\bigl(gy,F(v,u)\bigr) \\ &\qquad + a_9 q_2\bigl(gu,F(x,y)\bigr) + a_{10} q_2\bigl(gv,F(y,x)\bigr) \end{aligned} \tag{2.23}$$

holds for all $x,y,u,v \in X$. *Also, suppose we have the following hypotheses*:
(i) $F(X \times X) \subset g(X)$.
(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial metric* q_1.
Then the mappings F *and* g *have a coincidence point* (x,y) *satisfying* $gx = F(x,y) = F(y,x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof Given $x,y,u,v \in X$. It follows from (2.23) that

$$\begin{aligned} &q_1\bigl(F(x,y),F(u,v)\bigr) \\ &\quad \leq a_1 q_2(gx,gu) + a_2 q_2(gy,gv) + a_3 q_2\bigl(gx,F(x,y)\bigr) + a_4 q_2\bigl(gy,F(y,x)\bigr) \\ &\qquad + a_5 q_2\bigl(gu,F(u,v)\bigr) + a_6 q_2\bigl(gv,F(v,u)\bigr) + a_7 q_2\bigl(gx,F(u,v)\bigr) + a_8 q_2\bigl(gy,F(v,u)\bigr) \\ &\qquad + a_9 q_2\bigl(gu,F(x,y)\bigr) + a_{10} q_2\bigl(gv,F(y,x)\bigr) \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} &q_1\bigl(F(y,x),F(v,u)\bigr) \\ &\quad \leq a_1 q_2(gy,gv) + a_2 q_2(gx,gu) + a_3 q_2\bigl(gy,F(y,x)\bigr) + a_4 q_2\bigl(gx,F(x,y)\bigr) \end{aligned}$$

$$\begin{aligned}&+a_5q_2\big(gv,F(v,u)\big)+a_6q_2\big(gu,F(u,v)\big)\\&+a_7q_2\big(gy,F(v,u)\big)+a_8q_2\big(gx,F(u,v)\big)\\&+a_9q_2\big(gv,F(y,x)\big)+a_{10}q_2\big(gu,F(x,y)\big).\end{aligned}\tag{2.25}$$

Adding inequality (2.24) to inequality (2.25), we get

$$\begin{aligned}&q_1\big(q_1\big(F(x,y),F(u,v)\big)+F(y,x),F(v,u)\big)\\&\quad\le(a_1+a_2)\big[q_2(gx,gu)+q_2(gy,gv)\big]+(a_3+a_4)\big[q_2\big(gx,F(x,y)\big)+q_2\big(gy,F(y,x)\big)\big]\\&\qquad+(a_5+a_6)\big[q_2\big(gu,F(u,v)\big)+q_2\big(gv,F(v,u)\big)\big]\\&\qquad+(a_7+a_8)\big[q_2\big(gx,F(u,v)\big)+q_2\big(gy,F(v,u)\big)\big]\\&\qquad+(a_9+a_{10})\big[q_2\big(gu,F(x,y)\big)+q_2\big(gv,F(y,x)\big)\big].\end{aligned}\tag{2.26}$$

Therefore, the result follows from Theorem 2.1. □

Remark 2.2 If we take $q_1(x,y)=q_2(x,y)$ for all $x,y\in X$ and $a_7=a_8=a_9=a_{10}=0$, then Corollary 2.2 is reduced to Corollary 2.1 of Shatanawi and Pitea [38].

Corollary 2.3 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F:X\times X\to X$, $g:X\to X$ be two mappings. Suppose that there exists $k\in[0,1)$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2(gx,gu)+q_2(gy,gv)\big]\tag{2.27}$$

holds for all $x,y,u,v\in X$. Also, suppose we have the following hypotheses:

(i) $F(X\times X)\subset g(X)$.
(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a coincidence point (x,y) satisfying $gx=F(x,y)=F(y,x)=gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Remark 2.3 If we take $q_1(x,y)=q_2(x,y)$ for all $x,y\in X$, then Corollary 2.3 is reduced to Corollary 2.2 of Shatanawi and Pitea [38].

Corollary 2.4 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F:X\times X\to X$, $g:X\to X$ be two mappings. Suppose that there exists $k\in[0,1)$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2\big(gx,F(x,y)\big)+q_2\big(gy,F(y,x)\big)\big]\tag{2.28}$$

holds for all $x,y,u,v\in X$. Also, suppose we have the following hypotheses:

(i) $F(X\times X)\subset g(X)$.
(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a coincidence point (x, y) satisfying $gx = F(x, y) = F(y, x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u, u).

Remark 2.4 If we take $q_1(x, y) = q_2(x, y)$ for all $x, y \in X$, then Corollary 2.4 is reduced to Corollary 2.3 of Shatanawi and Pitea [38].

Corollary 2.5 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X \times X \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0, 1)$ such that the condition*

$$q_1\big(F(x,y), F(u,v)\big) + q\big(F(y,x), F(v,u)\big) \leq k\big[q_2\big(gu, F(u,v)\big) + q_2\big(gv, F(v,u)\big)\big] \tag{2.29}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X \times X) \subset g(X)$.*

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a coincidence point (x, y) satisfying $gx = F(x, y) = F(y, x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u, u).

Remark 2.5 If we take $q_1(x, y) = q_2(x, y)$ for all $x, y \in X$, then Corollary 2.5 is reduced to Corollary 2.4 of Shatanawi and Pitea [38].

Corollary 2.6 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X \times X \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0, \frac{1}{2})$ such that the condition*

$$q_1\big(F(x,y), F(u,v)\big) + q\big(F(y,x), F(v,u)\big) \leq k\big[q_2\big(gx, F(u,v)\big) + q_2\big(gy, F(v,u)\big)\big] \tag{2.30}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X \times X) \subset g(X)$.*

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a coincidence point (x, y) satisfying $gx = F(x, y) = F(y, x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u, u).

Corollary 2.7 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X \times X \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0, 1)$ such that the condition*

$$q_1\big(F(x,y), F(u,v)\big) + q\big(F(y,x), F(v,u)\big) \leq k\big[q_2\big(gu, F(x,y)\big) + q_2\big(gv, F(y,x)\big)\big] \tag{2.31}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X \times X) \subset g(X)$.

(ii) $g(X)$ *is a complete subspace of X with respect to the quasi-partial metric q_1*.

Then the mappings F and g have a coincidence point (x, y) satisfying $gx = F(x, y) = F(y, x) = gy$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u, u).

Let $g = I_X$ (the identity mapping) in Theorem 2.1 and Corollaries 2.1-2.7. Then we have the following results.

Corollary 2.8 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X \times X \to X$ be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0, 1)$ with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition*

$$\begin{aligned} &q_1\bigl(F(x,y),F(u,v)\bigr) + q_1\bigl(F(y,x),F(v,u)\bigr) \\ &\quad\leq k_1\bigl[q_2(x,u) + q_2(y,v)\bigr] + k_2\bigl[q_2\bigl(x,F(x,y)\bigr) + q_2\bigl(y,F(y,x)\bigr)\bigr] \\ &\qquad + k_3\bigl[q_2\bigl(u,F(u,v)\bigr) + q_2\bigl(v,F(v,u)\bigr)\bigr] + k_4\bigl[q_2\bigl(x,F(u,v)\bigr) + q_2\bigl(y,F(v,u)\bigr)\bigr] \\ &\qquad + k_5\bigl[q_2\bigl(u,F(x,y)\bigr) + q_2\bigl(v,F(y,x)\bigr)\bigr] \end{aligned} \tag{2.32}$$

holds for all $x, y, u, v \in X$. If (X, q_1) is a complete quasi-partial metric space, then the mapping F has a unique coupled fixed point of the form (u, u).

Corollary 2.9 *Let (X, q) be a complete quasi-partial metric space, $F : X \times X \to X$ be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0, 1)$ with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition*

$$\begin{aligned} &q\bigl(F(x,y),F(u,v)\bigr) + q\bigl(F(y,x),F(v,u)\bigr) \\ &\quad\leq k_1\bigl[q(x,u) + q(y,v)\bigr] + k_2\bigl[q\bigl(x,F(x,y)\bigr) + q\bigl(y,F(y,x)\bigr)\bigr] \\ &\qquad + k_3\bigl[q\bigl(u,F(u,v)\bigr) + q\bigl(v,F(v,u)\bigr)\bigr] + k_4\bigl[q\bigl(x,F(u,v)\bigr) + q\bigl(y,F(v,u)\bigr)\bigr] \\ &\qquad + k_5\bigl[q\bigl(u,F(x,y)\bigr) + q\bigl(v,F(y,x)\bigr)\bigr] \end{aligned} \tag{2.33}$$

holds for all $x, y, u, v \in X$. Then F has a unique coupled fixed point of the form (u, u).

Remark 2.6 Corollary 2.9 improve and extend Corollary 2.5 of Shatanawi and Pitea [38], the contractive condition is replaced by the new contractive condition defined by (2.35).

Corollary 2.10 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X \times X \to X$ be a mapping. Suppose that there exist $a_i \in [0, 1)$ ($i = 1, 2, 3, \ldots, 10$) with*

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + 2(a_7 + a_8) + a_9 + a_{10} < 1 \tag{2.34}$$

such that the condition

$$\begin{aligned} &q_1\big(F(x,y),F(u,v)\big) \\ &\quad\le a_1q_2(x,u)+a_2q_2(y,v)+a_3q_2\big(x,F(x,y)\big)+a_4q_2\big(y,F(y,x)\big) \\ &\qquad + a_5q_2\big(u,F(u,v)\big)+a_6q_2\big(v,F(v,u)\big)+a_7q_2\big(x,F(u,v)\big)+a_8q_2\big(y,F(v,u)\big) \\ &\qquad + a_9q_2\big(u,F(x,y)\big)+a_{10}q_2\big(v,F(y,x)\big) \end{aligned} \tag{2.35}$$

holds for all $x,y,u,v \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique coupled fixed point of the form (u,u).

Remark 2.7

(1) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$ and $a_7 = a_8 = a_9 = a_{10} = 0$, then Corollary 2.10 is reduced to Corollary 2.6 of Shatanawi and Pitea [38].
(2) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$ and $a_i = 0$ $(i = 3,4,5,\ldots,10)$, then Corollary 2.10 extends Theorem 2.1 of Aydi [12] on the class of quasi-partial metric spaces.
(3) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$, $a_1 = a_2$ and $a_i = 0$ $(i = 3,4,5,\ldots,10)$, then Corollary 2.10 extends the Corollary 2.2 of Aydi [12] on the class of quasi-partial metric spaces.
(4) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$ and $a_i = 0$ $(i = 1,2,4,6,7,8,9,10)$, then Corollary 2.10 extends Theorem 2.4 of Aydi [12] on the class of quasi-partial metric spaces.
(5) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$ and $a_i = 0$ $(i = 1,2,3,4,5,6,8,10)$, then Corollary 2.10 extends Theorem 2.5 of Aydi [12] on the class of quasi-partial metric spaces.
(6) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$, $a_3 = a_9$ and $a_i = 0$ $(i = 1,2,4,5,6,7,8,10)$, then Corollary 2.10 extends Corollary 2.6 of Aydi [12] on the class of quasi-partial metric spaces.
(7) If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$, $a_7 = a_9$ and $a_i = 0$ $(i = 1,2,3,4,5,6,8,10)$, then Corollary 2.10 extends Corollary 2.7 of Aydi [12] on the class of quasi-partial metric spaces.

Corollary 2.11 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X \times X \to X$ be a mapping. Suppose that there exists $k \in [0,1)$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big) + q\big(F(y,x),F(v,u)\big) \le k\big[q_2(x,u)+q_2(y,v)\big] \tag{2.36}$$

holds for all $x,y,u,v \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique coupled fixed point of the form (u,u).

Remark 2.8 If we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$, then Corollary 2.11 is reduced to Corollary 2.7 of Shatanawi and Pitea [38].

Corollary 2.12 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X \times X \to X$ be a mapping. Suppose that there exists $k \in [0,1)$ such that the* *condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2\big(x,F(x,y)\big)+q_2\big(y,F(y,x)\big)\big] \tag{2.37}$$

holds for all $x,y,u,v\in X$. If (X,q_1) is a complete quasi-partial metric space, then the mapping F has a unique coupled fixed point of the form (u,u).

Remark 2.9 If we take $q_1(x,y)=q_2(x,y)$ for all $x,y\in X$, then Corollary 2.12 is reduced to Corollary 2.8 of Shatanawi and Pitea [38].

Corollary 2.13 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F:X\times X\to X$ be a mapping. Suppose that there exists $k\in[0,1)$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2\big(u,F(u,v)\big)+q_2\big(v,F(v,u)\big)\big] \tag{2.38}$$

holds for all $x,y,u,v\in X$. If (X,q_1) is a complete quasi-partial metric space, then the mapping F has a unique coupled fixed point of the form (u,u).

Remark 2.10 If we take $q_1(x,y)=q_2(x,y)$ for all $x,y\in X$, then Corollary 2.13 is reduced to Corollary 2.9 of Shatanawi and Pitea [38].

Corollary 2.14 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F:X\times X\to X$ be a mapping. Suppose that there exists $k\in[0,\frac{1}{2})$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2\big(x,F(u,v)\big)+q_2\big(y,F(v,u)\big)\big] \tag{2.39}$$

holds for all $x,y,u,v\in X$. If (X,q_1) is a complete quasi-partial metric space, then the mapping F has a unique coupled fixed point of the form (u,u).

Corollary 2.15 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F:X\times X\to X$ be a mapping. Suppose that there exists $k\in[0,1)$ such that the condition*

$$q_1\big(F(x,y),F(u,v)\big)+q\big(F(y,x),F(v,u)\big)\le k\big[q_2\big(u,F(x,y)\big)+q_2\big(v,F(y,x)\big)\big] \tag{2.40}$$

holds for all $x,y,u,v\in X$. If (X,q_1) is a complete quasi-partial metric space, then the mapping F has a unique coupled fixed point of the form (u,u).

Now, we introduce an example to support our results.

Example 2.1 Let $X=[0,1]$, and two quasi-partial metrics q_1, q_2 on X be given as

$$q_1(x,y)=|x-y|+x \quad \text{and} \quad q_2(x,y)=\frac{1}{2}\big(|x-y|+x\big)$$

for all $x, y \in X$. Also, define $F : X \times X \to X$ and $g : X \to X$ as

$$F(x,y) = \frac{x+y}{16} \quad \text{and} \quad gx = \frac{x}{2}$$

for all $x, y \in X$. Then

(1) (X, q_1) is a complete quasi-partial metric space.
(2) $F(X \times X) \subset X$.
(3) F and g is w-compatible.
(4) For any $x, y, u, v \in X$, we have

$$q_1\big(F(x,y), F(u,v)\big) + q_1\big(F(y,x) + F(v,u)\big) \le \frac{1}{2}\big(q_2(gx,gu) + q_2(gy,gv)\big).$$

Proof The proofs of (1), (2), and (3) are clear. Next we show that (4). In fact, for $x, y, u, v \in X$, we have

$$\begin{aligned}
&q_1\big(F(x,y), F(u,v)\big) + q_1\big(F(y,x) + F(v,u)\big) \\
&\quad = q_1\left(\frac{x+y}{16}, \frac{u+v}{16}\right) + q_1\left(\frac{y+x}{16}, \frac{v+u}{16}\right) \\
&\quad = \frac{1}{8}\big(\big|x+y-(u+v)\big| + (x+y)\big) \\
&\quad = \frac{1}{4}\left(\left|\frac{1}{2}(x+y) - \frac{1}{2}(u+v)\right| + \frac{1}{2}(x+y)\right) \\
&\quad \le \frac{1}{4}\left(\left|\frac{1}{2}x - \frac{1}{2}u\right| + \frac{1}{2}x + \left|\frac{1}{2}y - \frac{1}{2}v\right| + \frac{1}{2}y\right) \\
&\quad = \frac{1}{2}\big(q_2(gx,gu) + q_2(gy,gv)\big).
\end{aligned}$$

Thus, F and g satisfy all the hypotheses of Corollary 2.3. So, F and g have a unique common coupled fixed point. Here $(0,0)$ is the unique common coupled fixed point of F and g. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this work. Both authors read and approved the final manuscript.

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